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Teaching Networks in General Mathematics Units 1 -4

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Introduction

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General Mathematics Study Design

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Graphs and networks – Unit 2.

This topic includes: (Undirected Networks)

- introduction to the notations, conventions and representations of types and properties of graphs, including edge, loop, vertex, the degree of a vertex, isomorphic and connected graphs and the adjacency matrix
- description of graphs in terms of faces (regions), vertices and edges and the application of Euler's formula for planar graphs
- connected graphs: walks, trails, paths, cycles and circuits with practical applications
- weighted graphs and networks, and an introduction to the shortest path problem (solution by inspection only) and its practical application
- trees and minimum spanning trees, greedy algorithms and their use to solve practical problems.

General Mathematics Study Design

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Networks and decision mathematics – Unit 4

This topic includes Undirected Networks (some overlap from Unit 2)

- the concepts, conventions and terminology of graphs including planar graphs and Euler's rule, and directed (digraphs) and networks
- use of matrices to represent graphs, digraphs and networks and their application.
- the concepts, conventions and notations of walks, trails, paths, cycles and circuits
- Eulerian trails and Eulerian circuits: the conditions for a graph to have an Eulerian trail or an Eulerian circuit, properties and applications
- Hamiltonian paths and cycles: properties and applications.
- trees and spanning trees, minimum spanning trees & Prim's algorithm
- use of minimal spanning trees to solve minimal connector problems.
- determination of the shortest path between two specified vertices in a graph, digraph or network by inspection
- Dijkstra's algorithm and its use to determine the shortest path between a given vertex and each of the other vertices in a weighted graph or network.

General Mathematics Study Design

This topic includes: (Directed Graphs – not covered at all in Unit 2)

- use of networks to model flow problems: capacity, sinks and sources
- solution of small-scale network flow problems by inspection and the use of the 'maximum-flow minimum-cut' theorem to aid the solution of larger scale problems.
- use of a bipartite graph and its tabular or matrix form to represent a matching problem
- determination of the optimum assignment/s of people or machines to tasks by inspection or by use of the Hungarian algorithm for larger scale problems.
- construction of an activity network from a precedence table (or equivalent) including the use of dummy activities where necessary
- use of forward and backward scanning to determine the earliest starting times (EST) and latest starting times (LST) for each activity
- use of earliest starting times and latest starting times to identify the critical path in the network and determine the float times for non-critical activities
- use of crashing to reduce the completion time of the project or task being modelled.

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NETWORKS

New to networks?

- Take the time to read and learn the theory. For many staff it will be entirely new! A few different textbooks can help.
- It's a unit that will require a bit of work if you haven't studied it before.
- There are many new terms in the module (path, trail, circuit, connected network, complete graph, spanning tree etc). Without knowing the content the VCAA exam questions will seem foreign.
- It is not a topic I would recommend learning by doing exam questions and trying to fill in the blanks by using the answers.

New to Networks

- Make glossary lists of all the different terms.
- It is a great topic to have pre-prepared notes that you share with students. You can either have students fill in the blanks or have examples for students to work through with you in class.
- Show lots of examples of different networks to illustrate the different terms that students will be introduced too (walks, trails, circuits etc).
- Learn about the undirected networks first (Unit 2) and then jump into directed networks (Unit 4)

Unit 2 General Mathematics

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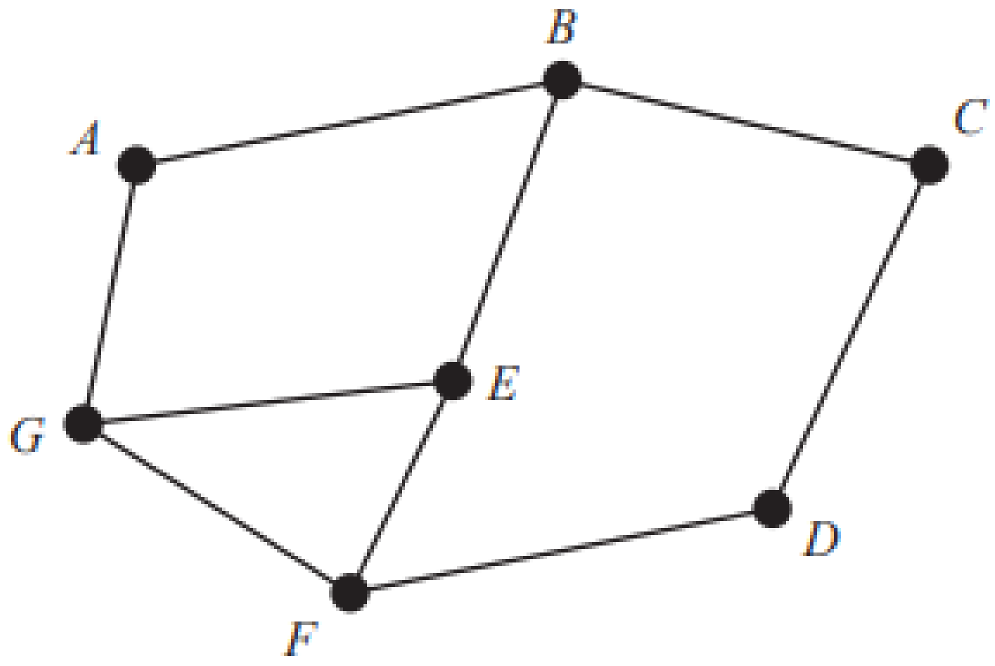
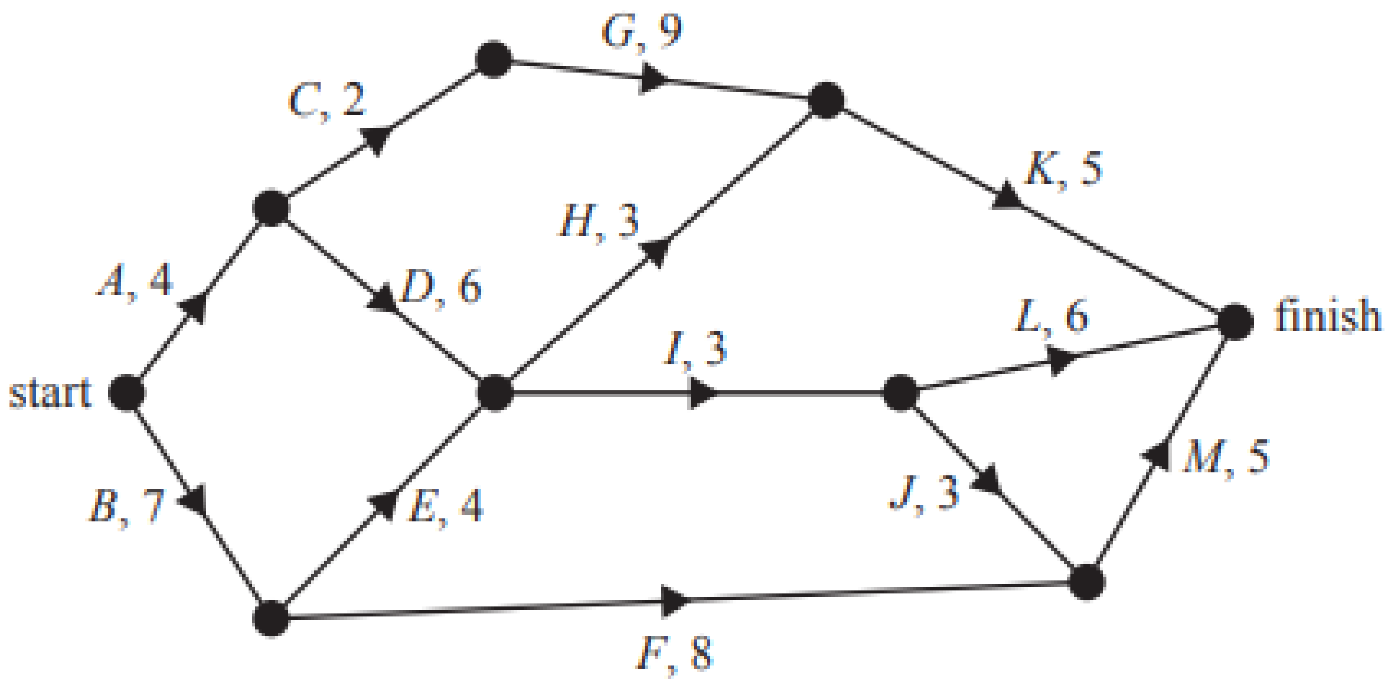
Undirected Networks

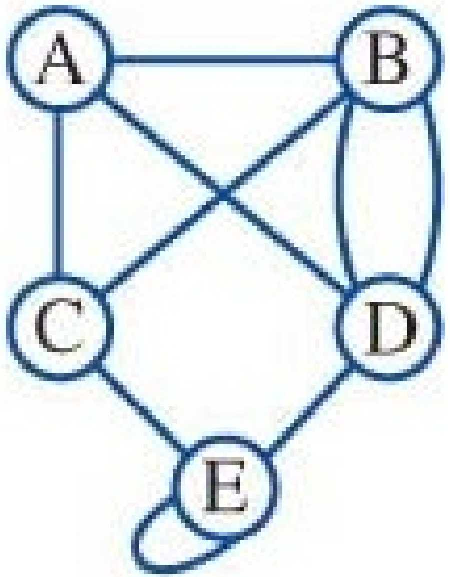


Directed Networks



Matrix Representation of Networks

- To represent the network, write the names of the vertices above the columns of the matrix and to the left side of the rows of the matrix. The number of edges/paths between the relevant vertices is then listed in the matrix.



$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 2 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

Note that E to E = 1.

This is because there is only path from E to E represented by the loop.

Whilst a loop counts as 2 when calculating the degree of a vertex it only counts as 1 edge/path.

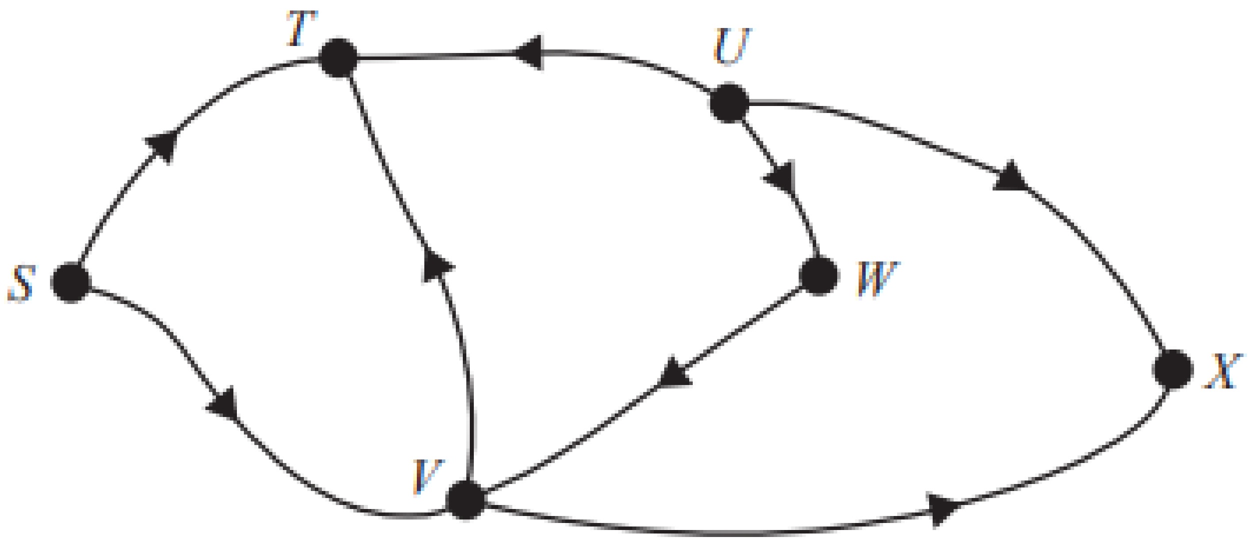
Consider introducing in Units 1/2

These topics are not part of the Unit 2 course but are highly recommended to discuss in Unit 2.

- Prim's algorithm for finding the minimum spanning tree
- Introduce Euler trails & circuits and Hamilton paths & cycles.
- Very simple introduction to directed graphs.

Introducing Directed Networks

The directed graph below represents a series of one-way streets.
The vertices represent the intersections of these streets.



The number of vertices that can be reached from S is

From S vertices, T, V and X (via V) can be reached. Therefore 3 vertices.

Introducing Directed Graphs

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- Ask students to write an adjacency matrix for the directed network.
- Ask students to list what routes are possible from each vertex.
- Ask students True/False questions about the network

(E.g. True/False – Can you travel from vertex S to vertex W?)

A good understanding of simple directed networks and being able to explain different aspects of the directed graphs (like the questions above) will make Unit 4 content more manageable for students.

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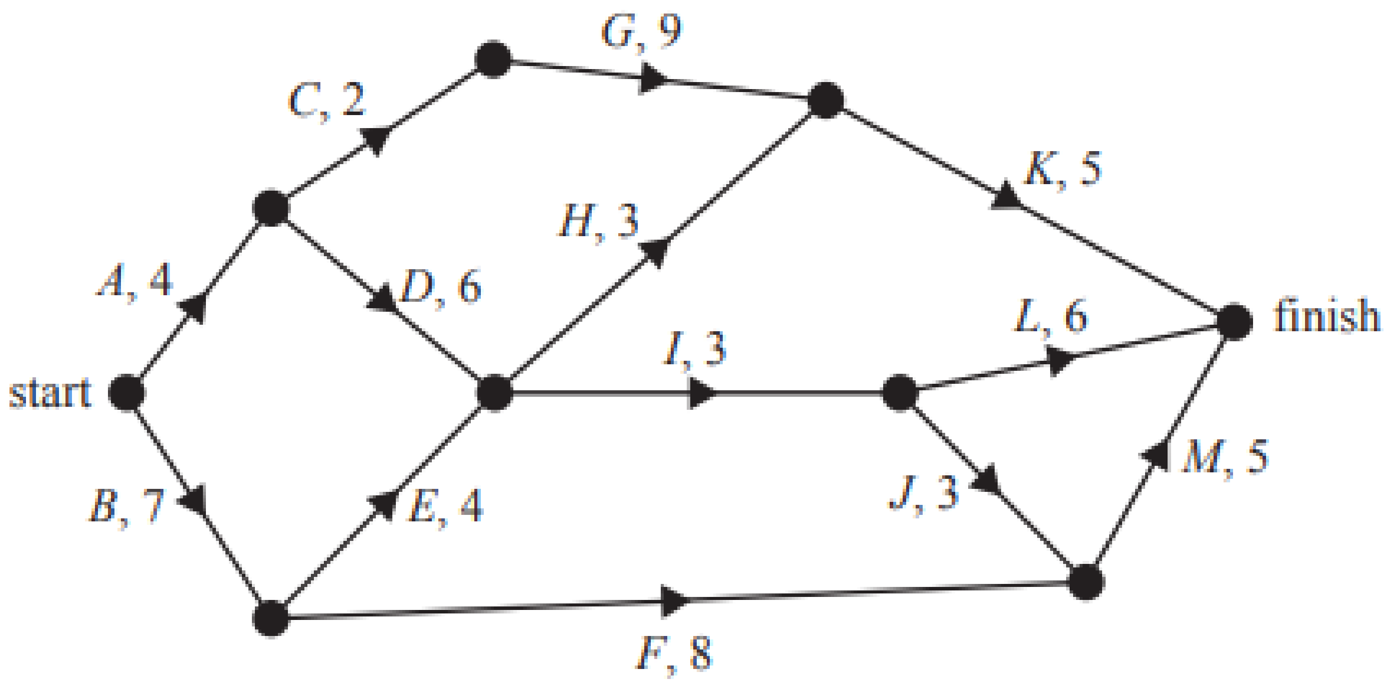
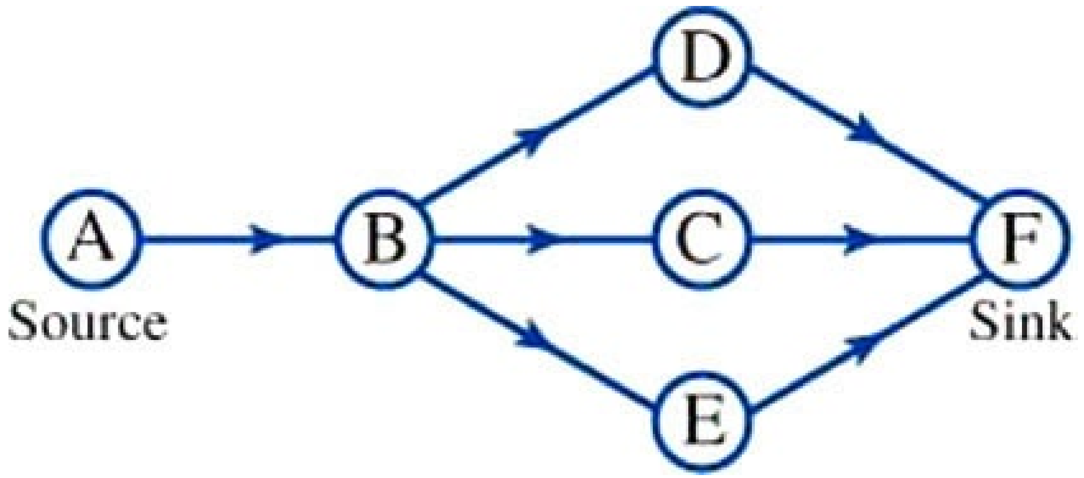
Directed Networks



Flow Capacities and Maximum Flow

The network's starting node(s) is called the *source.* This is where all flows commence. The flow goes through the network to the end node(s) which is called the *sink.*

The *flow capacity* (capacity) of an edge is the amount of flow that an edge can allow through if it is not connected to any other edges.



Minimum Cut & Maximum Flow

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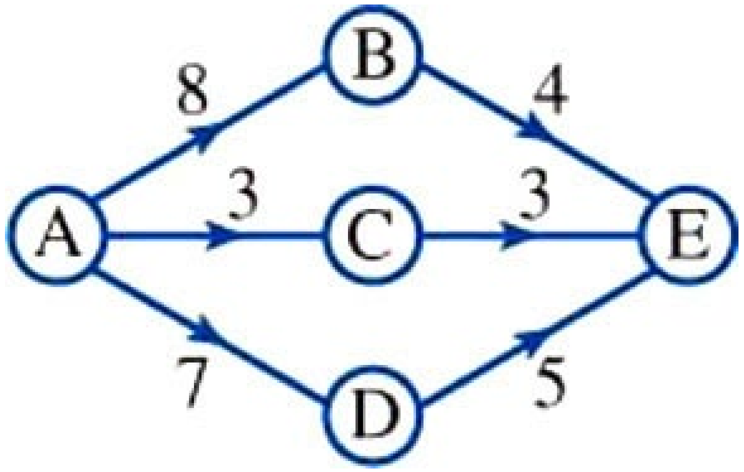
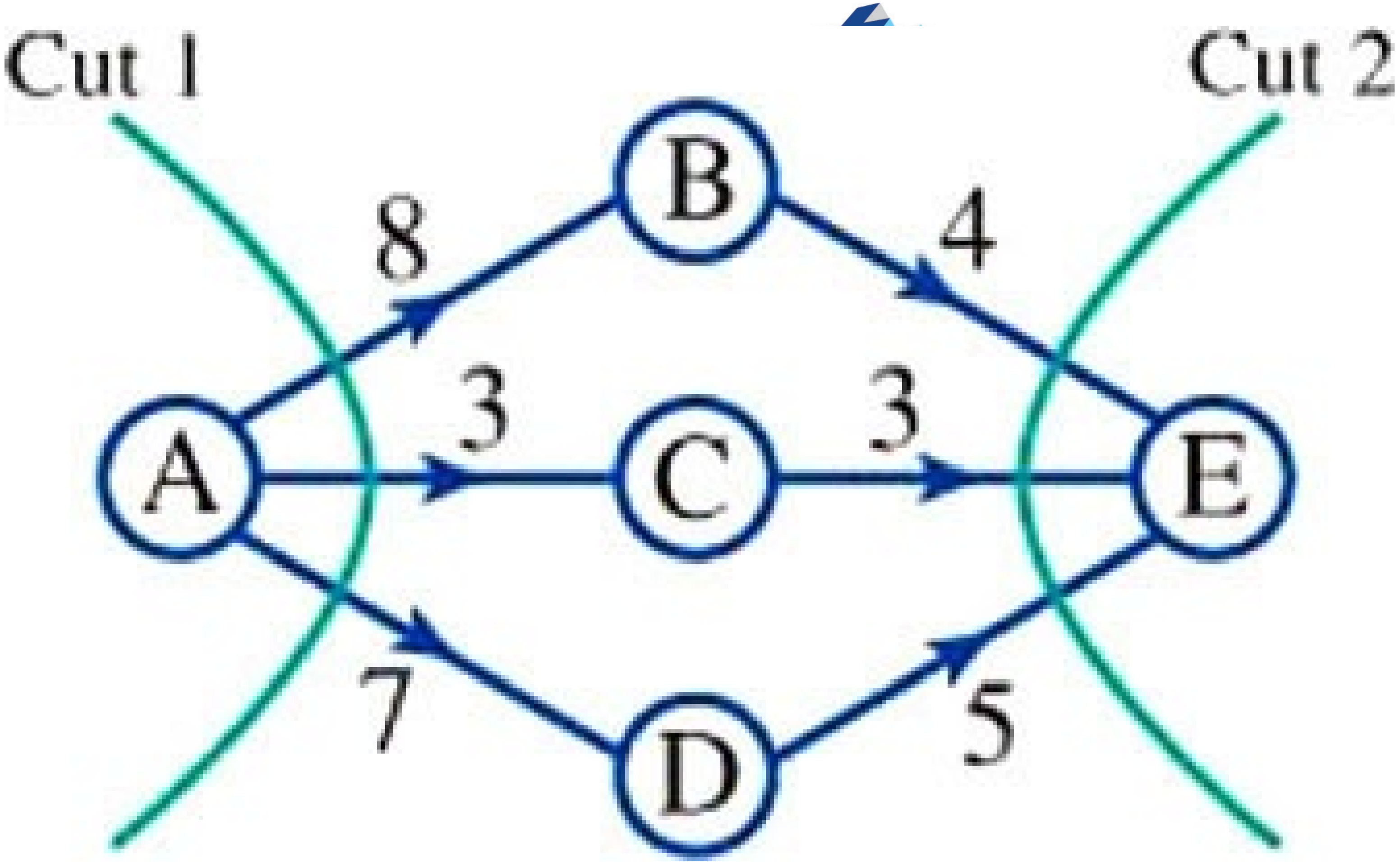
- To determine the maximum flow, the network first needs to be divided or 'cut' into two parts.
- A cut in a network diagram is a line drawn through a number of edges which stops *all* flow from the source to the sink

- The value of the cut is the total flow of the edges that are cut.
- The *minimum cut* is the cut with the minimum value.
- The ***maximum flow*** through a network is equal to the value of the ***minimum cut.***

MAXIMUM FLOW = MINIMUM CUT





In this network to find the maximum flow we need to cut off the source (A) from the sink (E)

Cut 1	Cut 2	Cut 3	Cut 4	Cut 5	Cut 6
18	12	14	14	16	16

Minimum Cut = 12, therefore maximum flow = 12

Value of a cut

Ensuring that all cuts have been made is a complicated procedure.

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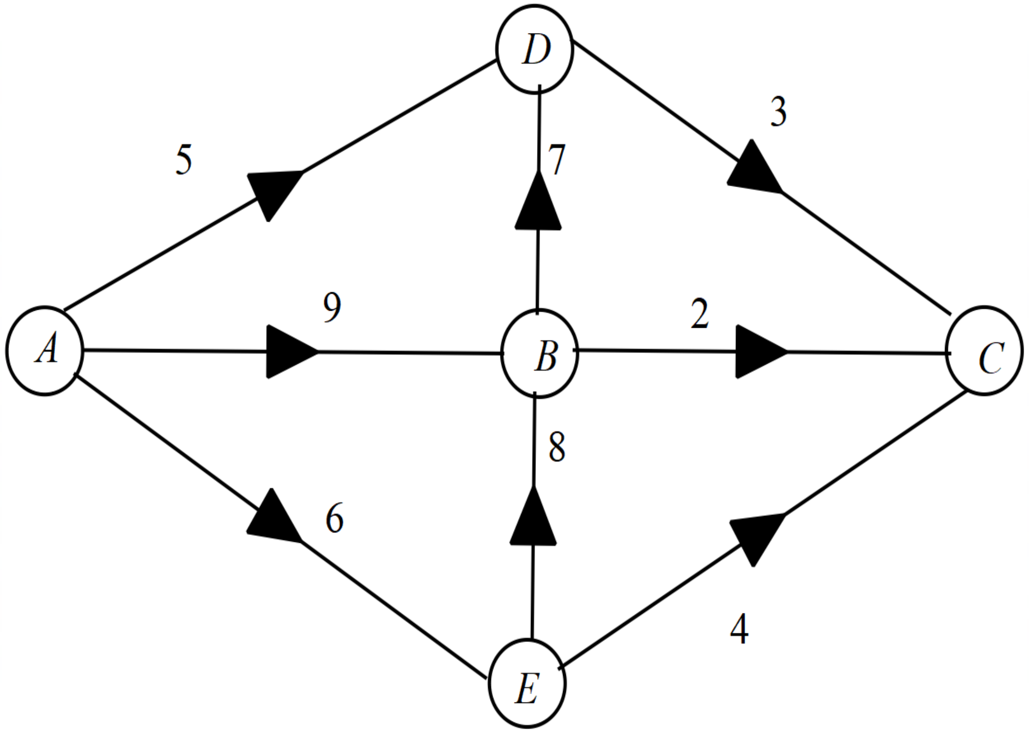
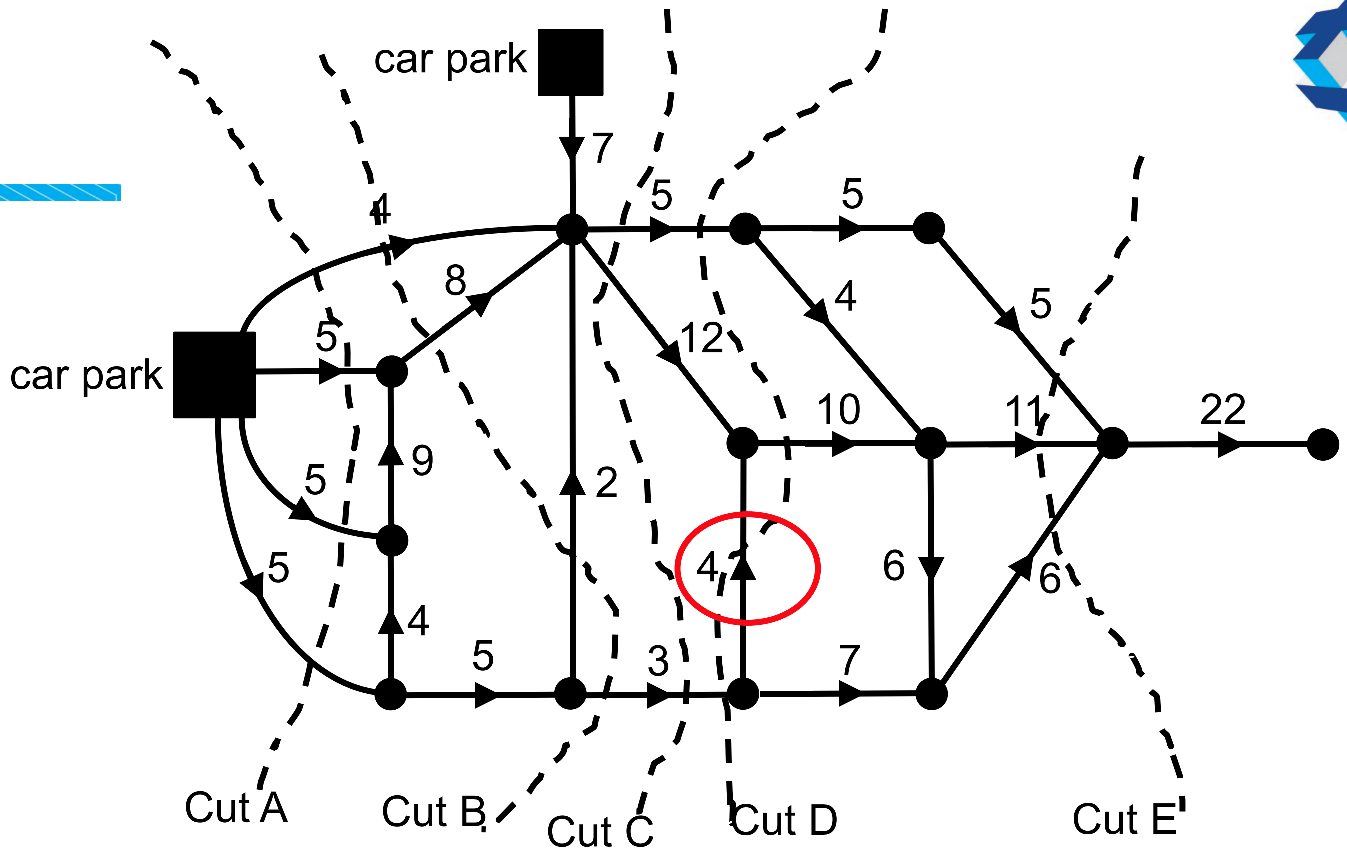


Sometimes an edge that the cut passes through does not contribute to the source being disconnected to the sink and therefore the flow value can be ignored when calculating the value of the cut.

Cut 1 passes through AE (6), AB (9), **BD (7)** and CD (3)
However, the edge BD with a value of 7 is going the wrong direction and does not need to be counted for the source to be disconnected from the sink,
Cut 1 = 3 + 9 + 6 = 18

Cut 2 passes through AE (6), **BE (8)**, BC (2), DB (7) & AD (5)
However, the edge BE is going the wrong direction.
Cut 2 = 6 + 2 + 7 + 5 = 20

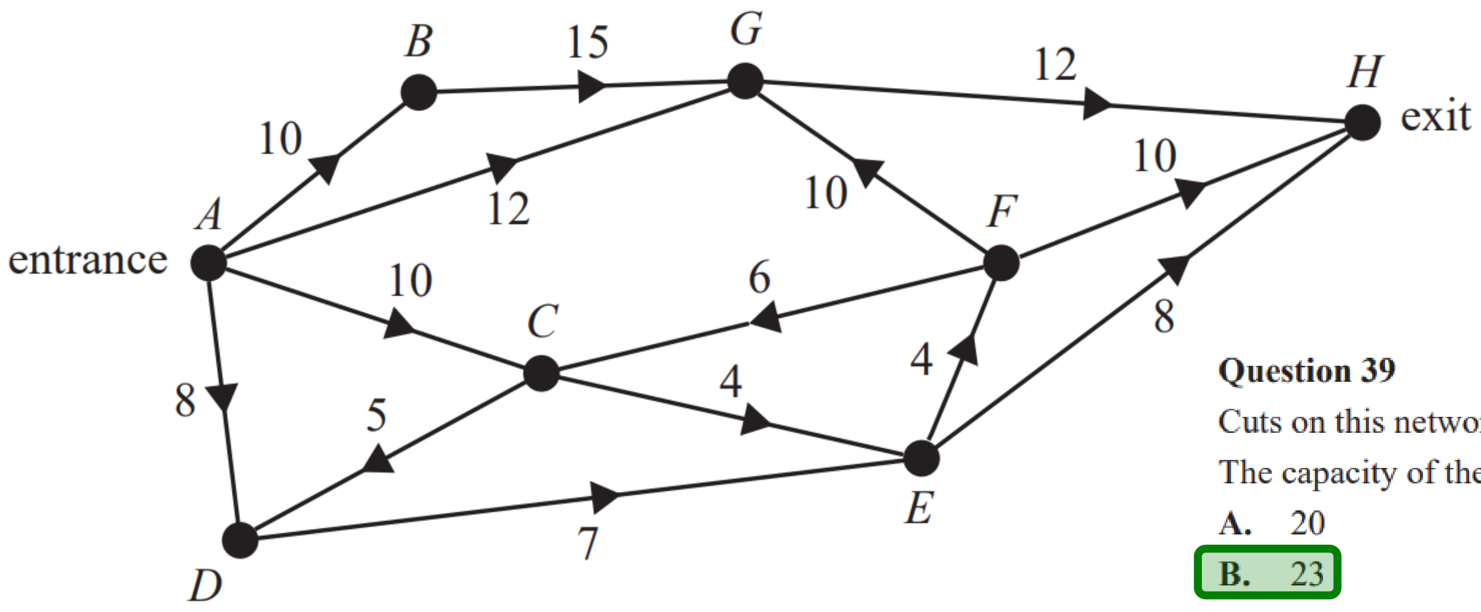
2014 Exam 1 Q9



A.	Cut A	4%	DOES NOT separate both sources from sink
B.	Cut B	39%	DOES NOT separate both sources from sink
C.	Cut C	9%	5 + 12 + 3 = 20
D.	Cut D	23%	5 + 10 + 3 = 18, 4 in wrong direction
E.	Cut E	24%	5 + 11 + 6 = 22

2023 Exam 1 Q39

The network below shows the one-way paths between the entrance, A, and the exit, H, of a children's maze.
The vertices represent the intersections of the one-way paths.
The number on each edge is the maximum number of children who are allowed to travel along that path per minute.



Question 39

Cuts on this network are used to consider the possible flow of children through the maze.

The capacity of the minimum cut would be

A. 20

B. 23

C. 24

D. 29

E. 30

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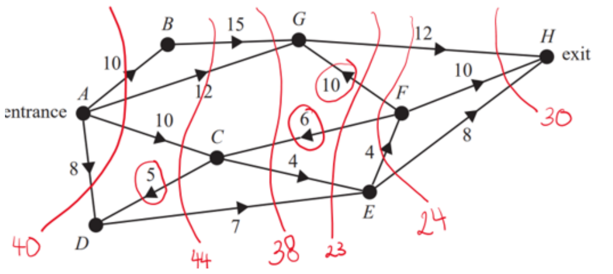
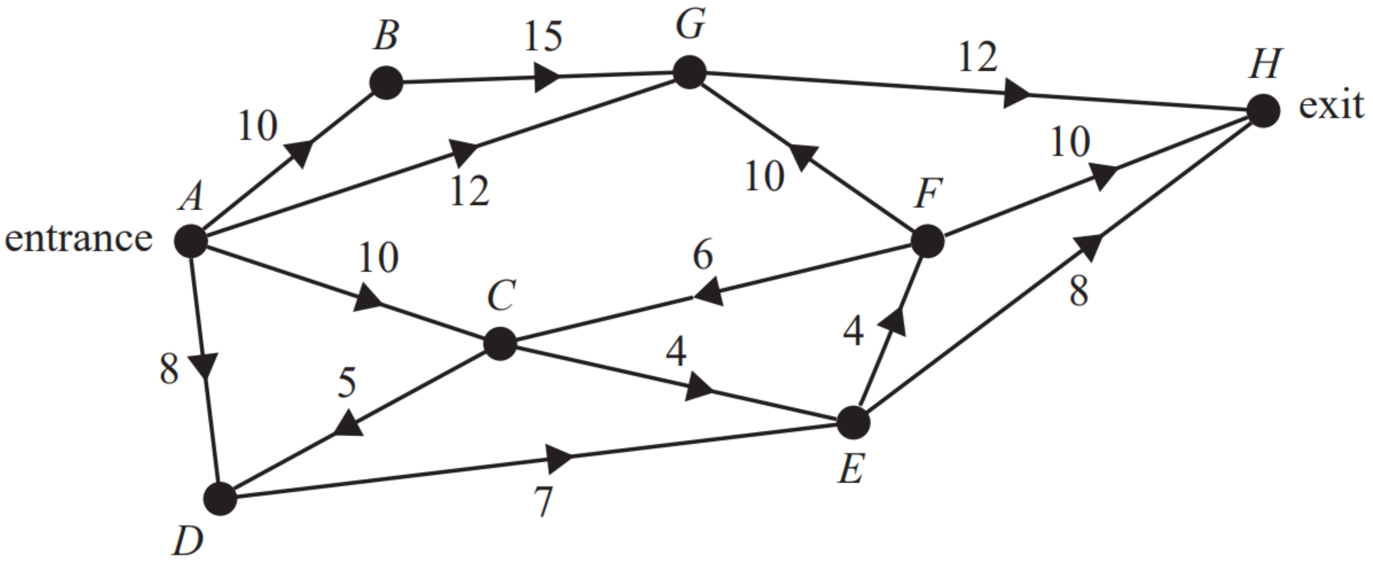
Min cut/ max flow= 23





Question 40

One path in the maze is to be changed.

Which one of these five changes would lead to the largest increase in flow from entrance to exit?

A. increasing the capacity of flow along the edge CE to 12 — Would increase max flow by 1 (min cut =24)

B. increasing the capacity of flow along the edge FH to 14 — No change to min cut/max flow

C. increasing the capacity of flow along the edge GH to 16 — Would increase max flow by 4 (min cut = 27)

D. reversing the direction of flow along the edge CF — Would increase max flow by 6 (min cut 29)

E. reversing the direction of flow along the edge GF — Would increase max flow by 7 (min cut 30)

Critical Path Analysis

Imagine you need to cook chicken, potatoes and carrots for dinner.

The potatoes take 60 minutes in the oven.
The chicken takes 35 minutes in the oven.
The carrots take 10 minutes on the stove top.

How long will it take to cook the dinner?

Critical Path Analysis

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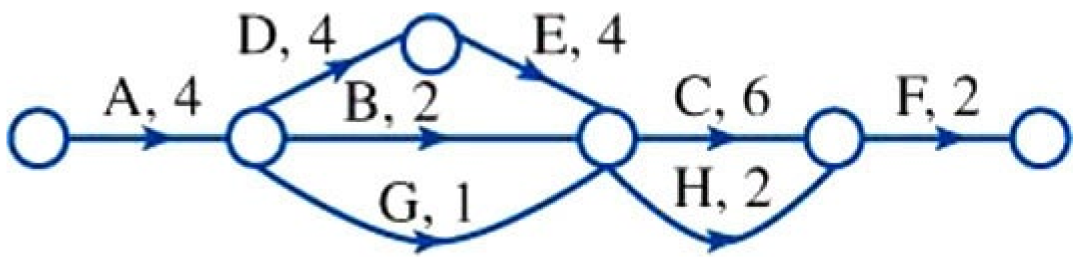
Critical Path Analysis is used to determine shortest completion time for a project or schedule as well as earliest start times, latest start times and any activities that can be delayed.

Activity letter	Activity	Predecessor	Time (min)
A	Prepare breakfast	—	4
B	Cook breakfast	A	2
C	Eat breakfast	B, E, G	6
D	Have shower	A	4
E	Get dressed	D	4
F	Brush teeth	C, H	2
G	Download email	A	1
H	Read email	B, E, G	2
Total time			25



2023 Exam 1 Q 38

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Question 38

A particular building project has ten activities that must be completed.

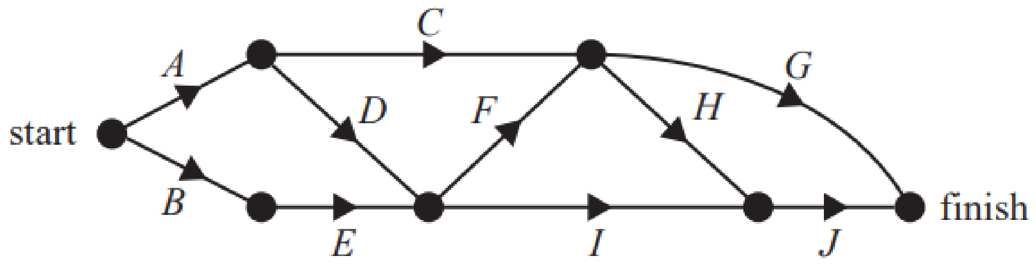
These activities and their immediate predecessor(s) are shown in the table below.

Activity	Immediate predecessor(s)
A	–
B	–
C	A
D	A
E	B
F	D, E
G	C, F
H	F
I	D, E
J	H, I

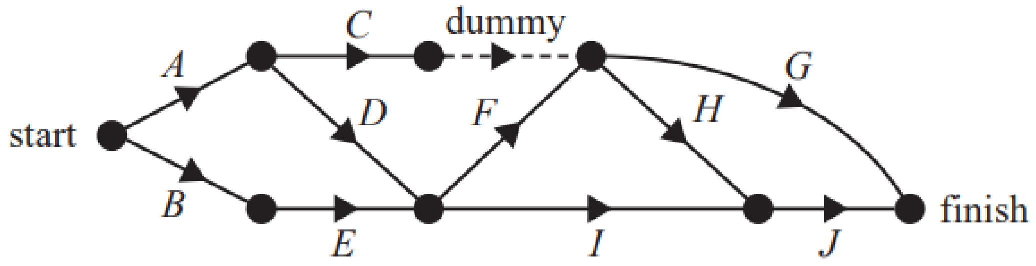
A directed graph that could represent this project is

2023 Exam 1 Q 38

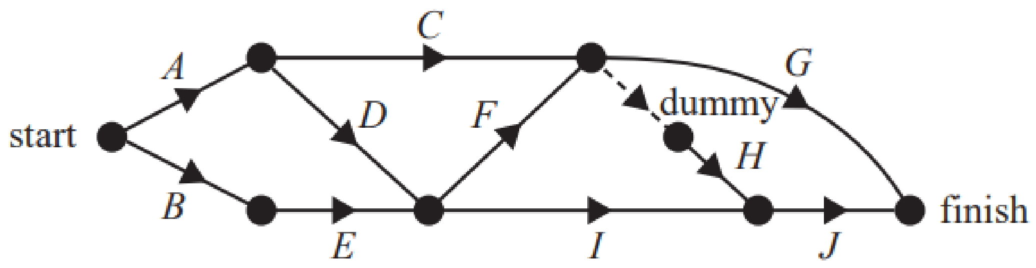
A.



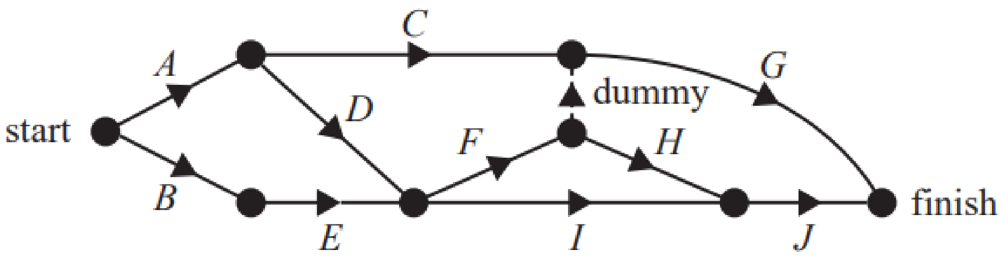
B.



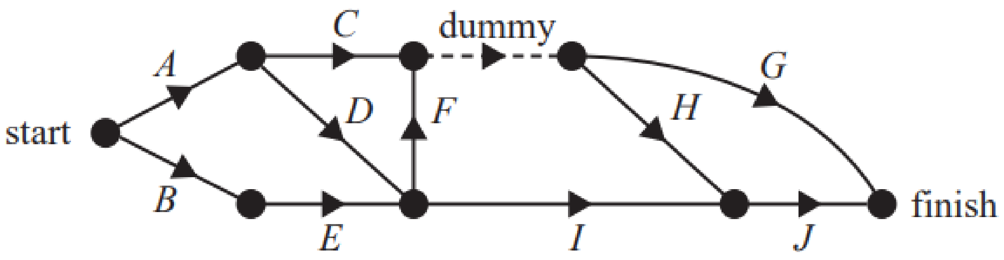
C.



D.



E.



Forward Scanning

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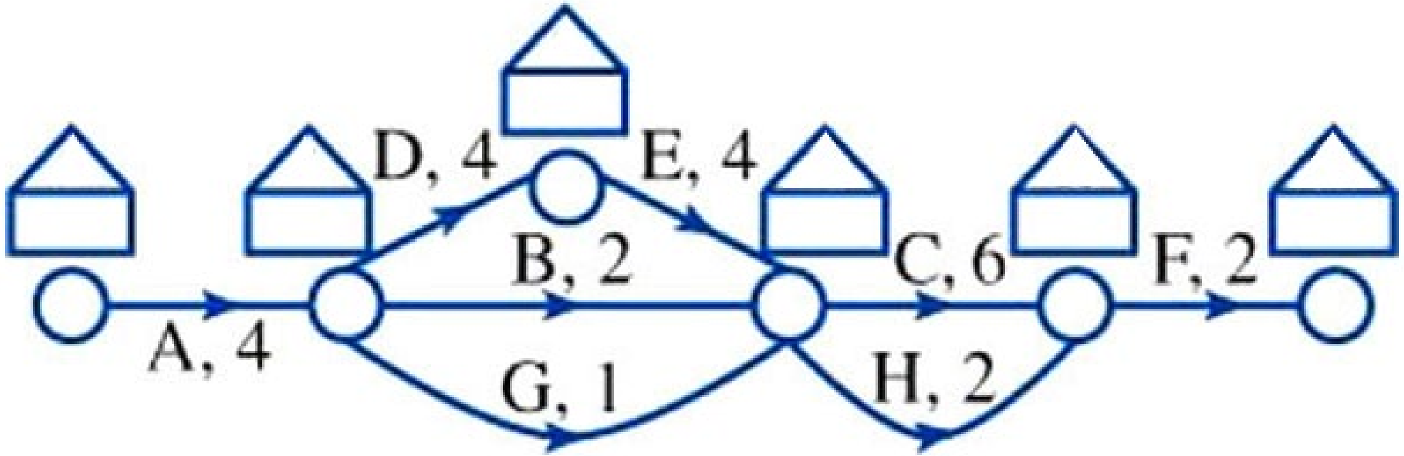
By forward scanning through a network we can calculate the earliest start times for each activity and the earliest completion time for the whole project. The *earliest start time* (EST) is the earliest that any activity can be started after all prior activities have been completed.

Draw a small 'house' at each vertex. The EST will be written in the top triangle (roof).

By forward scanning through a network we can calculate the earliest start times for each activity and the earliest completion time for the whole project. The *earliest start time* (EST) is the earliest that any activity can be started after all prior activities have been completed.
The EST is determined by looking at all the previous activities, starting with the immediate predecessors and working back to the start of the project.



Forward Scanning

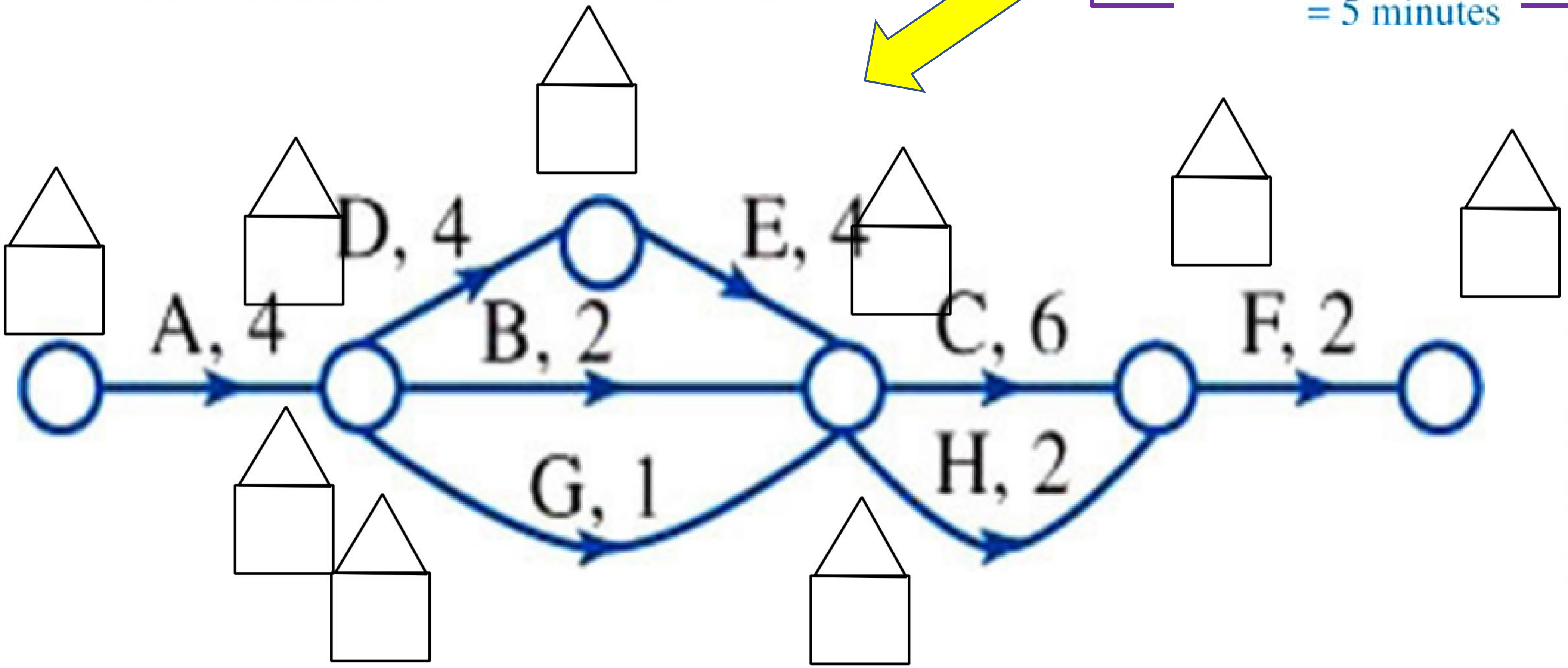
$$\text{A–D–E} = 4 + 4 + 4 = 12 \text{ minutes}$$

Calc… all the paths… largest value… e appr…

$$\text{A–B} = 4 + 2 = 6 \text{ minutes}$$

$$\text{A–G} = 4 + 1 = 5 \text{ minutes}$$

I recommend drawing a ‘house’ at the beginning of each edge. This is so students can see what the EST is for each activity.



Backward Scanning

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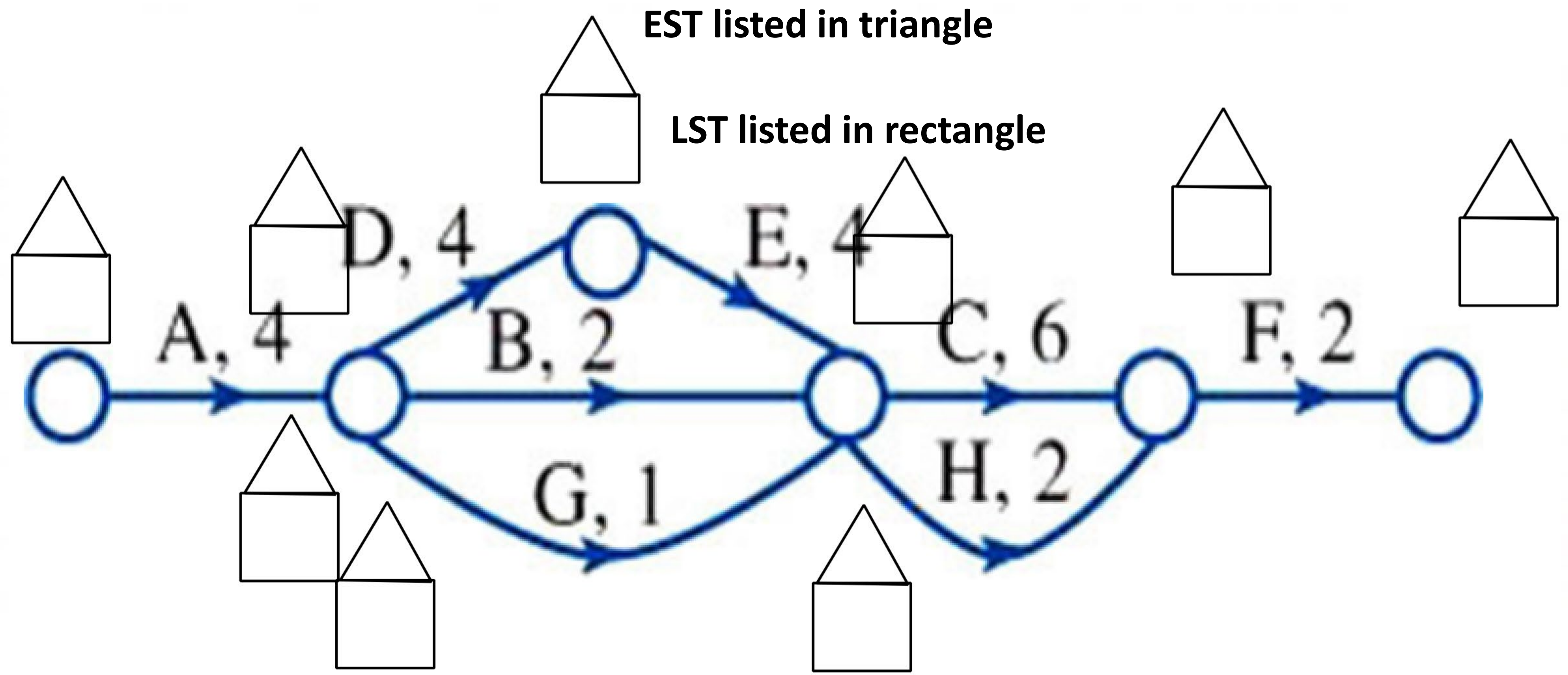
To complete critical path analysis, a procedure called *backward scanning* must be performed.

- *Backward scanning* starts at the end node and moves backward through the network subtracting the time of each edge from the earliest start time of each succeeding node.
- When two or more paths are followed back to the same node the smallest value is recorded.
- The results of each backward scanning step yield the *latest start time* (LST) for each activity.
- Latest start time is the latest time an activity can start without delaying the project.

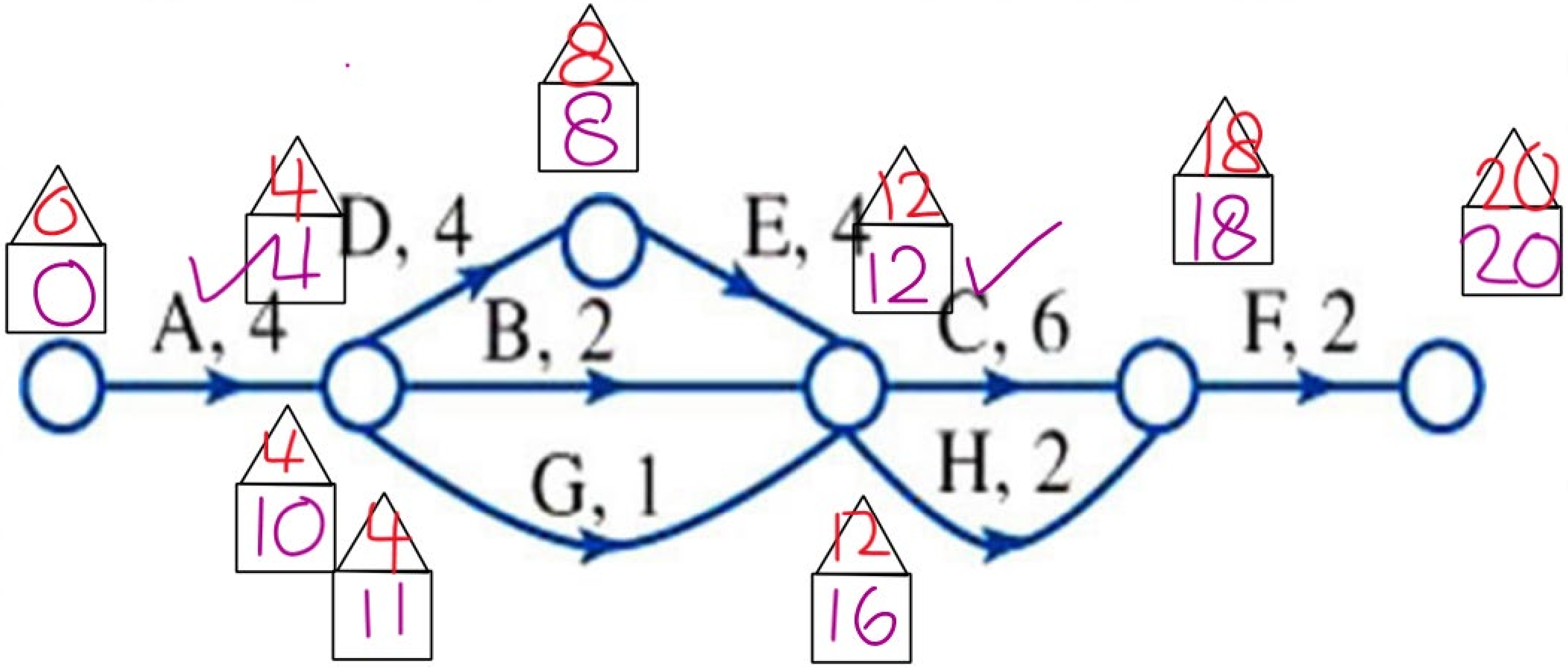
Backward Scanning



Critical Path

The path through the network which follows those activities that cannot be delayed without causing the entire project to be delayed is called the *critical path.*

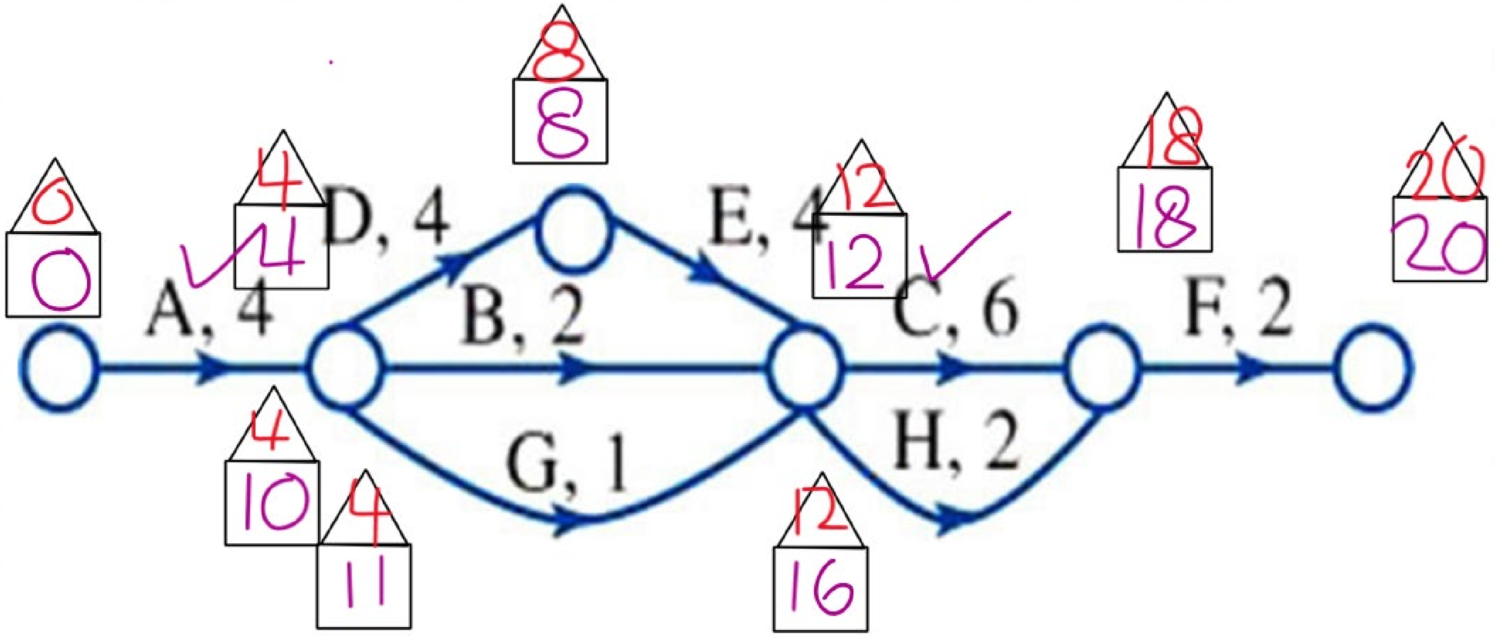
Therefore, the critical path for the activities listed in network would be A–D–E–C–F. This path takes the longest time (20 minutes). The critical path will always be the longest path!



Float time

Float time, **also called 'slack', is the maximum time that an activity can be delayed without delaying a subsequent activity on the critical path and thus affecting the earliest completion time.**

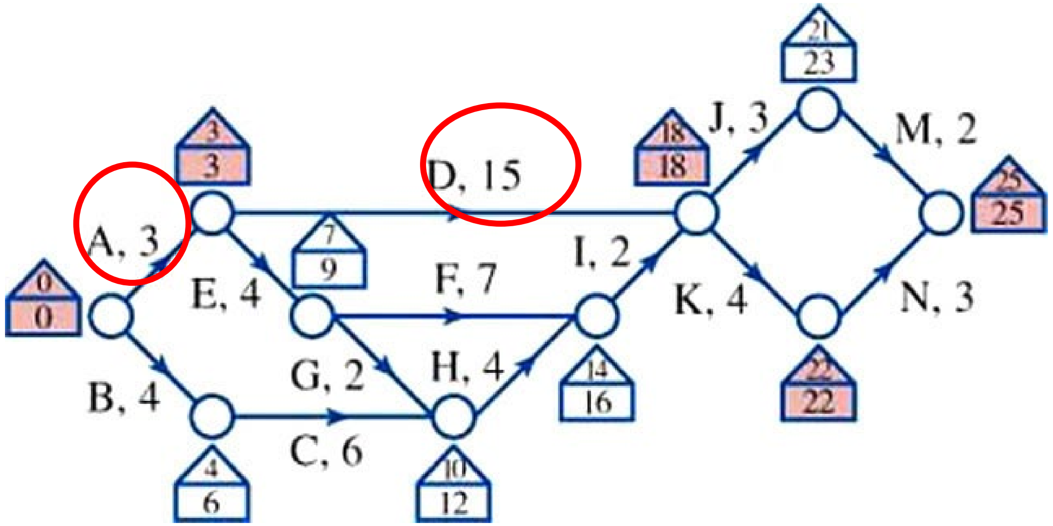
Activities that are NOT on the critical path will have a float time greater than 0.
Float time = LST – EST for each activity.



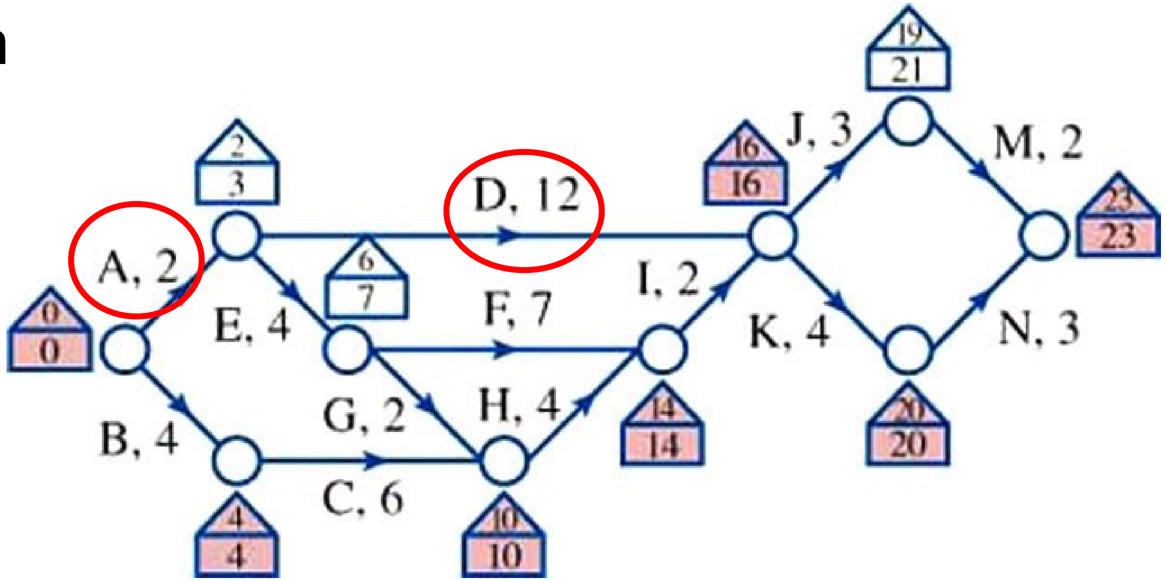
Activity	Float Time
A	0
B	6
C	0
D	0
E	0
F	0
G	7
H	4

Crashing

If the manager employed extra workers for a critical activity, its duration time could be reduced, hence reducing the completion time for the project. The reduction in the duration time of an activity is called *crashing.*
Crashing may result in a different critical path



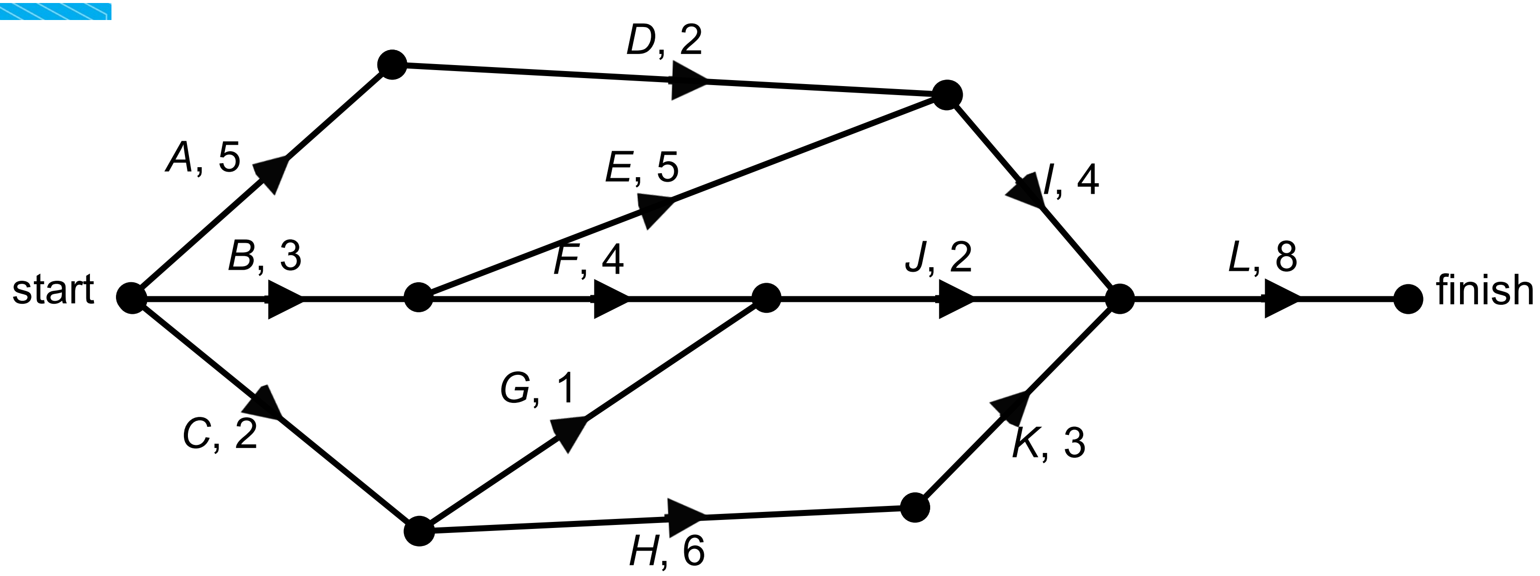
Critical Path = ADKN (25 days)



Critical Path = BCHIKN (23 days)
Reducing A by 1 day & D by 3 days
has resulted in a 2 day overall reduction.

2016 Exam 1 Q7

The directed graph below shows the sequence of activities required to complete a project. All times are in hours.



There is one critical path for this project.

Three critical paths would exist if the duration of activity

A. *I* were reduced by two hours.

B. *E* were reduced by one hour.

C. *G* were increased by six hours.

D. *K* were increased by two hours.

E. *F* were increased by two hours.

2016 Exam 1 Q7

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This table shows the five possible paths, with their original lengths and after each option is applied (critical path).

Path	length	A. $I-2$	B. $E-1$	C. $G+6$	D. $K+2$	E. $F+2$
A-D-I-L	19	17	19	19	19	19
B-E-I-L	20	18	19	20	20	20
B-F-J-L	17	17	17	17	17	19
C-G-J-L	13	13	13	19	13	13
C-H-K-L	19	19	19	19	21	19

There is one critical path for this project.
Three critical paths would exist if the duration of activity

A. *I* were reduced by two hours. 13%

B. *E* were reduced by one hour. 45%

C. *G* were increased by six hours. 14%

D. *K* were increased by two hours. 12%

E. *F* were increased by two hours. 15%

2023 Exam 2 Q14

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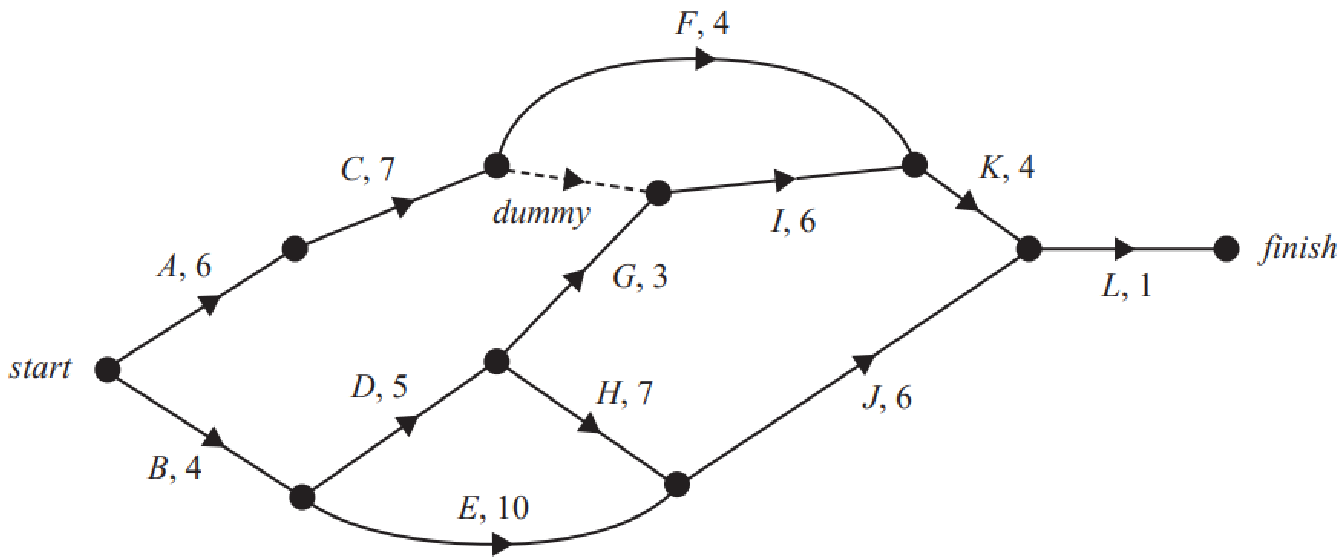
Question 14 (5 marks)

One of the landmarks in state A requires a renovation project.

This project involves 12 activities, A to L. The directed network below shows these activities and their completion times, in days.



The table below shows the 12 activities that need to be completed for the renovation project.

It also shows the earliest start time (EST), the duration, and the immediate predecessors for the activities.

The immediate predecessor(s) for activity I and the EST for activity J are missing.

Activity	EST	Duration	Immediate predecessor(s)
A	0	6	–
B	0	4	–
C	6	7	A
D	4	5	B
E	4	10	B
F	13	4	C
G	9	3	D
H	9	7	D
I	13	6	
J		6	E, H
K	19	4	F, I
L	23	1	J, K

2023 Exam 2 Q14

a. Write down the immediate predecessor(s) for activity I.

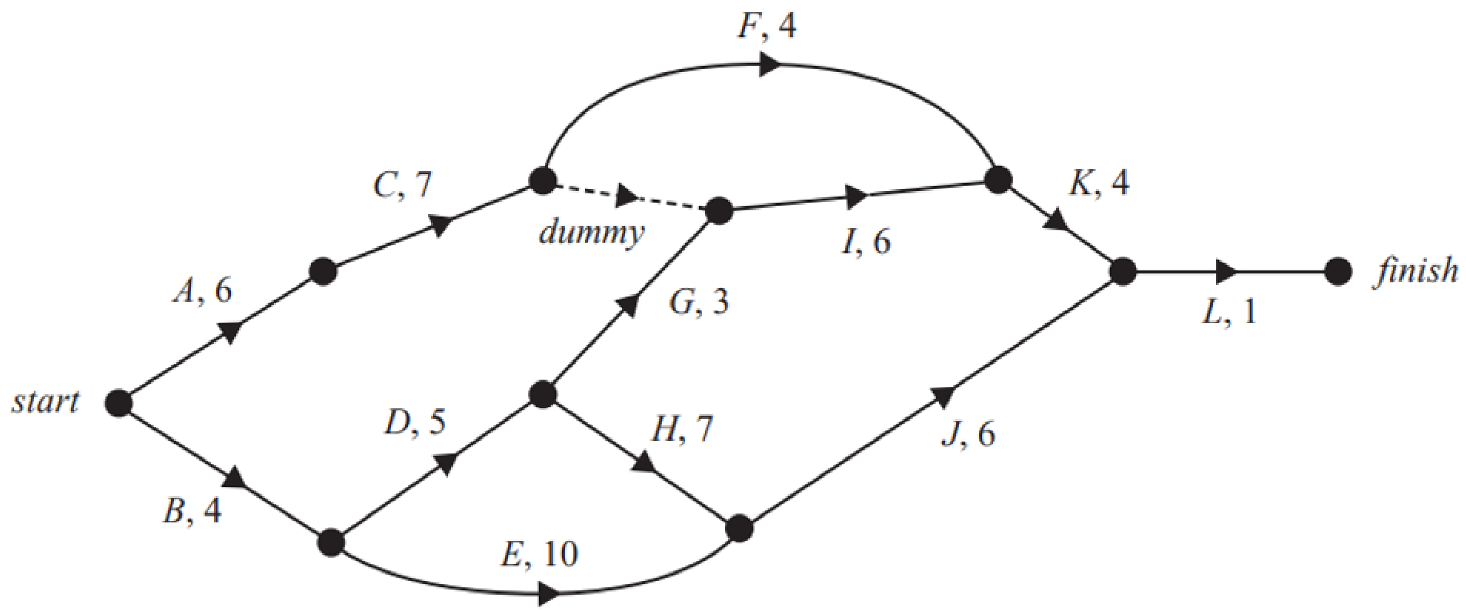
C & G (be aware of the dummy activity)

b. What is the earliest start time, in days, for activity J ?

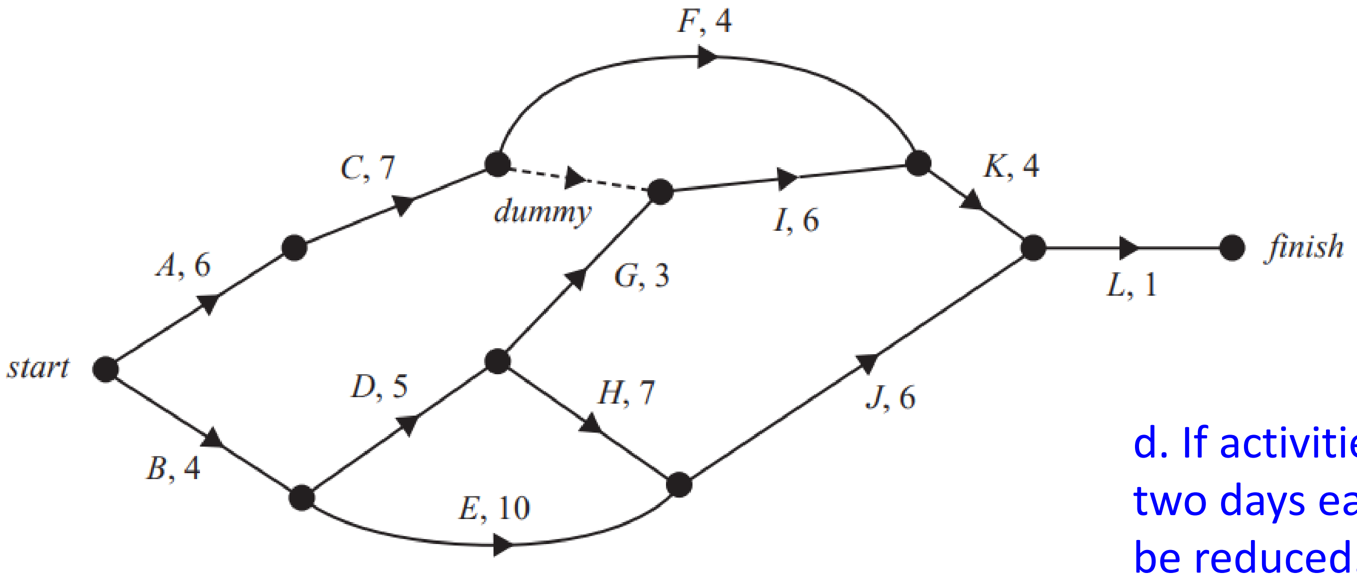
4 + 5 + 7 = 16 $(B \rightarrow D \rightarrow H)$

c. How many activities have a float time of zero?

5 – Activities A, C, I, K, L (all on the critical path)



2023 Exam 2 Q14



Path	Length	A reduced by 2	B reduced by 2
ACFKL	22	20	20
ACIKL	24	22	22
BDGIKL	23	23	21
BDHJL	23	23	21
BEJL	21	21	19

d. If activities A and B have their completion time reduced by two days each, the overall completion time of the project will be reduced. What will be the maximum reduction time, in days?

Maximum reduction time is 2 days.

Many students answered 22 days (providing the new reduced completion time)

2023 Exam 2 Q14

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The managers of the project are able to reduce the time, in days, of six activities. These reductions will result in an increase in the cost of completing the activity. The maximum decrease in time of any activity is two days.

Activity	*A*	*B*	*F*	*H*	*I*	*K*
Daily cost ($)	1500	2000	2500	1000	1500	3000

The managers of the project have a maximum budget of $15000 to reduce the time for several activities to produce the maximum reduction in the project's overall completion time.

Complete the table, showing the reductions in individual activity completion times that would achieve the earliest completion time within the $15000 budget.

Activity	**Reduction in completion time (0, 1 or 2 days)**
A	
B	
F	
H	
I	
K	

2023 Exam 2 Q14

Activity	*A*	*B*	*F*	*H*	*I*	*K*
Daily cost ($)	1500	2000	2500	1000	1500	3000

Path	**Length**	**A -2**	**B -2**	**I - 2**	**H-2**	**K-1**
ACFKL	22	20	20	20	20	19
ACIKL	24	22	22	20	20	19
BDGIKL	23	23	21	19	19	18
BDHJL	23	23	21	21	19	19
BEJL	21	21	19	19	19	19
		$3000	$4000	$3000	$2000	$3000

Activity	**Reduction in completion time (0, 1 or 2 days)**
A	
B	
F	
H	
I	
K	

Mathematical Investigation – Units 1/2

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This comprises two weeks of investigation into one or two practical or theoretical contexts or scenarios based on content from areas of study and application of key knowledge and key skills for the outcomes. Investigation is to be incorporated in the development of concepts, skills and processes for the unit.

There are three components to investigation:

- Formulation: Overview of the context or scenario, and related background, including historical or contemporary background as applicable, and the mathematisation of questions, conjectures, hypotheses, issues or problems of interest
- Exploration: Investigation and analysis of the context or scenario with respect to the questions of interest, conjecture or hypotheses, using mathematical concepts, skills and processes, including the use of technology and application of computational thinking.
- Communication: Summary, presentation and interpretation of the findings from the investigation and related applications.

Networks Mathematical Investigation

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<u>Some possible contexts or scenarios for investigation in Unit 2/4 include:</u>

- Postal/rubbish truck routes of local area
- Map of camping ground and explore routes to different areas of interest.
- Local Olympic torch relay visiting areas of interest (use local area)
- Look at direct interstate flights to capital cities around Australia
- School bus route

Common Misconceptions with Networks – Unit 2

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- Students often have trouble drawing networks! They need more practise
- Difficulty labelling routes correctly as Euler trail/circuits or Hamilton paths/cycles. Indicates the need for better glossary notes and using their notes to answer questions. Also asking students to justify why the route is Euler/Hamilton may assist student understanding.
- Planar graphs – students need to know that graphs may be able to be redrawn to be planar (so no edges cross over each other).

Common Misconceptions with Networks – Unit 4

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- Students often have trouble drawing networks! They need more practise

- Difficulty in determining float times. Students can usually determine which activities have a float time greater than zero but have trouble determining the actual float times.

- Crashing – highlight for students by using a table to check for other paths which could become a new critical path.

- Forward/backward scanning. Be careful that students use the larger number in forward scanning and smaller number in backward scanning.

Glossary Page

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Build a glossary of terms (& images) with students for their bound reference.

- Connected Graph
- Completed Graph
- Euler’s Formula
- Tree, isolated vertex, loop, cycle, bipartite
- Degree of a vertex
- Minimum cut/ maximum flow
- Euler trail/circuit & Hamilton path/cycle
- EST, LST, Float times
- Critical Path

Video tutorials on Networks

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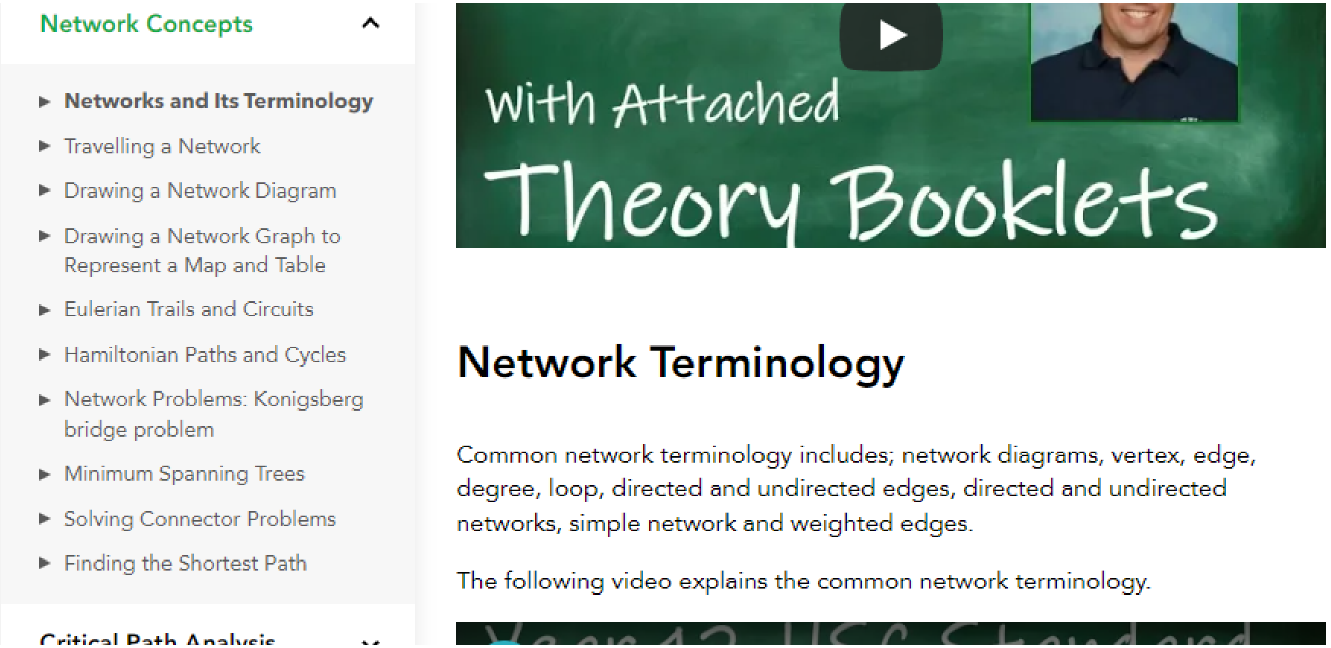
https://artofsmart.com.au/hsctogether/what-is-critical-path-analysis/



Any questions?

Feel free to contact me anytime

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Be in it to WIN!